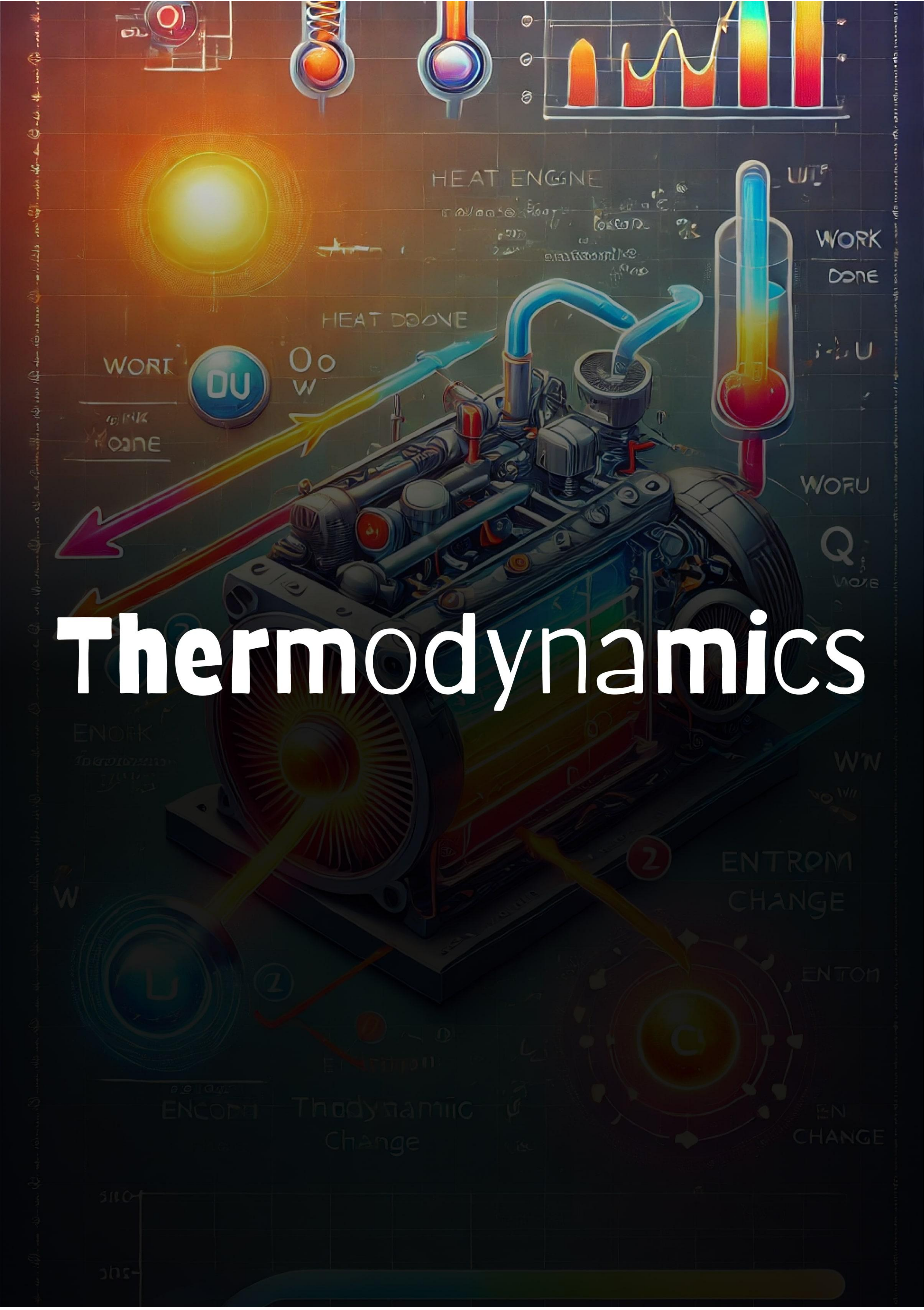


Thermodynamics

Thermodynamics

KINETIC THEORY OF GASES

IDEAL GAS

When we assume a gas to be ideal, we mean the following assumptions:

(i) Gas is sparsed (the volume occupied by gas is much much greater than volume of its molecules.)

(ii) The molecules of gas are in a random motion in all possible directions without any preferential direction.

(iii) The molecules of gas make perfectly elastic collisions with the walls of the container.

(iv) There is no force of interaction b/w the molecules of the gases.

IDEAL GAS EQUATION

$$PV = nRT$$

$$P = \frac{\rho RT}{M}$$

P = Pressure of gas (in Pascal)

V = volume of Container (in m^3)

n = no. of mole of gas

T = temp. of gas (in Kelvin)

ρ = density of gas (Kg/m^3)

M = molar mass of gas (in Kg/mol)

ROOT MEAN SQUARE OF A PHYSICAL QUANTITY

Root mean square of any physical quantity is the square root of the mean of the squares.

For instance, let v_i represents the velocity associated with i^{th} particle then root mean square velocity will be defined as

$$v_{rms} = \sqrt{\frac{\sum v_i^2}{N}}$$

RELATION B/W rms VELOCITY , PRESSURE & DENSITY

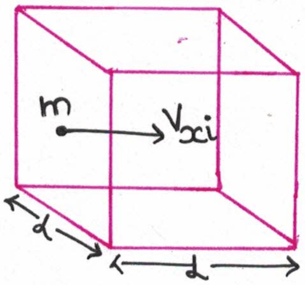
$n \rightarrow$ No. of moles

$N \rightarrow$ No. of particles

$m \rightarrow$ mass of 1 particle

$M \rightarrow$ Mass of 1 mol.

$$nM = Nm$$



V_{xi} = x component of velocity of ith particle.

$$J_{xi} = \Delta p_{xi} = 2m V_{xi} \quad \text{—①}$$

$$\Delta t = \frac{2d}{V_{xi}} \quad \text{—②}$$

$$F_{xi} = \frac{J_{xi}}{\Delta t}$$

$$F_{xi} = \frac{m V_{xi}^2}{d}$$

$$F_x = \frac{m}{d} \sum v_{xi}^2 \quad \text{—③}$$

$$\sum v_{xi}^2 = \sum v_{yi}^2 = \sum v_{zi}^2$$

$$= \sum \left(\frac{V_{xi}^2 + V_{yi}^2 + v_{zi}^2}{3} \right) = \sum \frac{v_i^2}{3}$$

$$\sum v_{xi}^2 = \frac{1}{3} \sum v_i^2 \quad \text{—④}$$

$$v_{rms} = \sqrt{\frac{\sum v_i^2}{N}}$$

$$\sum v_i^2 = N v_{rms}^2 \quad \text{—⑤}$$

$$\sum v_{xi}^2 = \frac{N}{3} v_{rms}^2 \quad \text{—⑥}$$

$$F_x = \frac{m}{d} \cdot \frac{N}{3} \cdot V_{rms}^2$$

$$P = \frac{F}{A} = \frac{F_x}{d^2}$$

$$P = \frac{m}{d^3} \times \frac{N}{3} v_{rms}^2$$

$mN \rightarrow$ Total mass in container

$d^3 \rightarrow$ volume of container

$$P = \frac{1}{3}\rho v_{rms}^2 \qquad - ⑦$$

$$\boxed{v_{rms} = \sqrt{\frac{3P}{\rho}}} \qquad - ⑧$$

Relation b/w rms Velocity & Temperature:

$$v_{rms} = \sqrt{\frac{3RT}{M}} \qquad \text{using ⑧}$$

$$P = \frac{\rho RT}{M}$$

Que) At what temp. will rms velocity of O_2 be same as that & of H_2 at room temperature.

$$\sqrt{\frac{3RT_{O_2}}{M_{O_2}}} = \sqrt{\frac{3RT_{H_2}}{M_{H_2}}}$$

$$T_{O_2} = 16 \times 298 = 4768 K$$

RELATION B/W KINETIC ENERGY (U) OF A MONOATOMIC IDEAL GAS & ITS TEMPERATURE

$$U = \sum \frac{1}{2} m v_i^2$$

$$U = \frac{1}{2} m \sum v_i^2$$

$$U = \frac{1}{2} m N v_{rms}^2 \qquad \text{using ⑤}$$

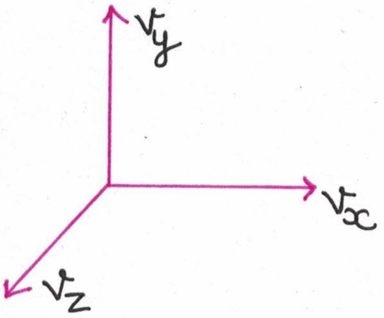
$$U = \frac{1}{2} n M \times \frac{3RT}{M}$$

$$U = \frac{3}{2} nRT$$

DEGREE OF FREEDOM (f)

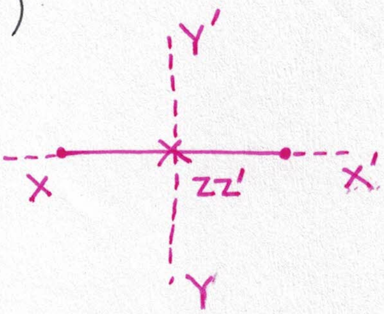
Minimum no. of variables required to specify the state of motion of a typical molecule's Kinetic energy is called degree of freedom (f).

1.) $f_{\text{mono atomic}} \longrightarrow 3\ (v_x, v_y, v_z)$

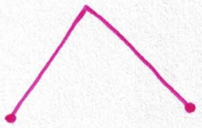


2.) $f_{diatomic/linear} \longrightarrow 5(v_x, v_y, v_z, w_{yy'}, w_{zz'})$

$w_{xx'} \neq$ is not included because moment of inertia is very less.



3.) $f_{poly/non\text{-}linear} \longrightarrow 6(v_x, v_y, v_z, W_{xx'}, W_{yy'}, W_{zz'})$



LAW OF EQUIPARTITION OF ENERGY

Energy associated with each degree of freedom is equal at any temperature.

$$\frac{\text{Energy of } n \text{ moles}}{f} = \frac{\cancel{3}/2\, nRT}{\cancel{3}}$$

$$\frac{\text{Energy}}{\text{d.o.f}} = \frac{1}{2} nRT$$

$$\frac{\text{Energy}}{\text{d.o.f mole}} = \frac{1}{2} RT$$

$$\frac{\text{Energy}}{\text{dof molecule}} = \frac{1}{2} \frac{R}{N_A} T = \frac{1}{2} KT$$

↳ boltzman Const.

$$\boxed{U = \frac{nfRT}{2}}$$

(for general gas)

MOLAR SPECIFIC INTERNAL ENERGY (C_v)

The change in internal energy of one mole of gas in the process of heating it by 1°C is called molar specific internal energy.

Mathematically,

$$\boxed{C_v = \frac{f}{2} R}$$

FIRST LAW OF THERMODYNAMICS

For any system,

$$dQ = dU + dW$$

Here dQ is the energy supplied to the system
du is the internal energy change of the system
dw is the work done by the system.

INTEGRAL FORM OF FIRST LAW

$$\Delta Q = \Delta U + \Delta W$$

POLYTROPIC PROCESS

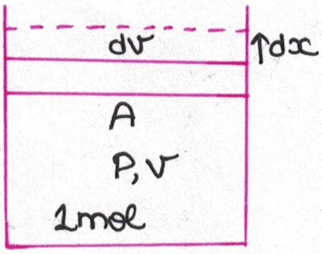
Any process of the form $Pv^x =$ constant represents a polytropic process. By replacing x by various numbers, we can generate the other standard processes namely,

$x = 0$	Isobaric
$x = 1$	Isothermal
$x = \gamma$	Adiabatic
$x = \infty$	Isochoric $(P^{1/\infty} v = \text{Con})$

MOLAR SPECIFIC WORK (Ω)

The work done by increasing the temperature of 1 mole of gas by 1°C is called molar specific work (Ω).

Consider, 1 mole of a gas contained in a vessel whose instantaneous volume is v and cross sectional area is A. Further, let dv be volume change because of piston shifting by dx. Then we can write small amount of work done by gas as



$$dv = A.dx$$

$$dw = F.dx$$

$$dw = \frac{F}{A} \times A dx$$

$$dw = P dv$$

$$\left(\frac{dw}{dT}\right)_{1mol} = \left(\frac{Pdv}{dT}\right)_{mol}$$

$$\boxed{\Omega = \left(\frac{Pdv}{dT}\right)_{1mol}}$$

MOLAR SPECIFIC WORK FOR POLYTROPIC PROCESS

$$Pv = RT \quad -①$$

$$Pv^{x} = C \quad -②$$

$$\ln P + \ln v = \ln R + \ln T \quad -③ \qquad [\text{from} ①]$$

differentiate ③

$$\frac{1}{P}dP + \frac{1}{v}dv = \frac{1}{T}dT \quad -④$$

$$\ln P + x\ln v = \ln C \quad -⑤$$

$$\frac{1}{P}dP + \frac{x}{v}dv = 0 \quad -⑥$$

④ - ⑥

$$(1-x)\frac{dv}{v} = \frac{dT}{T}$$

$$\frac{dv}{dT} = \frac{1}{1-x}\cdot\frac{v}{T}$$

$$P.\frac{dv}{dT} = \frac{1}{(1-x)}\cdot\frac{Pv}{T}$$

$$\frac{Pdv}{dT} = \frac{R}{(1-x)}$$

$$\boxed{\Omega = \frac{R}{(1-x)}}$$

MOLAR SPECIFIC HEAT

The amount of heat required to raise temperature of 1mole of gas by 1degree celsius.

$$C = \left(\frac{dQ}{dT}\right)_{mol}$$

RELATION B/w C_Ω & C_v

$$dQ = dw + du$$

$$\frac{dQ}{dT} = \frac{dw}{dT} + \frac{du}{dT}$$

$$\left(\frac{dQ}{dT}\right)_{1mol} = \left(\frac{dw}{dT}\right)_{1mol} + \left(\frac{du}{dT}\right)_{1mol}$$

$$C = \Omega + C_v$$

$$\boxed{C_x = \frac{R}{1-x} + \underbrace{\frac{f}{2}R}_{C_v}}$$

NOTE: $nR\Delta T \longrightarrow P_f v_f - P_i v_i$

For any Polytropic process:

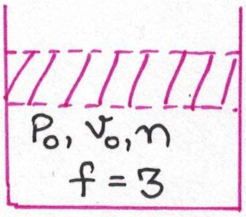
$$\Delta U = \frac{nf}{2} R\Delta T \longrightarrow (P_f v_f - P_i v_i)\frac{f}{2}$$

$$\Delta W = \frac{nR}{(1-x)}\Delta T \longrightarrow \frac{(P_f v_f - P_i v_i)}{(1-x)}$$

$$\Delta Q = n\left(\frac{R}{1-x} + \frac{fR}{2}\right)\Delta T \longrightarrow (P_f v_f - P_i v_i)\left(\frac{1}{1-x} + \frac{f}{2}\right)$$

Que.) n moles of a monoatomic gas are confined in a vessel to undergo a process $Pv^{-1} =$ const. Find the ΔQ, ΔU and ΔW in the process of doubling the volume.

$$\frac{P_0}{v_0} = \frac{P_2}{2v_2}$$



$$v_f = 2v_0$$

$$P_f = 2P_0$$

$$\Delta U = \frac{f}{2} Rn\Delta T$$

$$\Delta U = \frac{f}{2}(P_f v_f - P_i v_i)$$

$$= \frac{3}{2}(3P_0 v_0) = \frac{9P_0 v_0}{2}$$

$$\Delta W = \frac{3P_0 v_0}{2}$$

$$\Delta Q = 6P_0 v_0$$

Que.) In the previous set up if instead of doubling the volume we were to double the temp., then what would be the final temp. Also calculate the corr. ΔU, ΔW, ΔQ

$$P_0 v_0 = nRT_0$$

$$T_0 = \frac{P_0 v_0}{nR}$$

$$\frac{P_f}{v_f} = \frac{P_0}{v_0}$$

$$T_f = 2T_0 = \frac{2P_0V_0}{nR}$$

$$\Delta U = \frac{nf}{2} R\Delta T$$

$$= \cancel{n}\frac{3}{2}\cancel{R}\left(\frac{P_0V_0}{\cancel{nR}}\right)$$

$$\Delta U = \frac{3}{2}P_0V_0$$

$$\Delta W = \frac{nR}{1-x}(\Delta T)$$

$$= \frac{\cancel{nR}}{2}\left(\frac{P_0V_0}{\cancel{nR}}\right) = \frac{P_0V_0}{2}$$

$$\Delta Q = 2P_0V_0$$

Que.) Suppose the total heat supplied to the system is 1000J. Find ΔW & ΔU. Set up is as like as in previous que.

$$\frac{\Delta W}{\Delta Q} = \frac{1}{(1-x)+\left(\frac{1}{1-x}+\frac{f}{2}\right)} = \frac{2}{2+f(1-x)} = \frac{1}{4}$$

$$\Delta W = 25\Delta J$$

$$\Delta U = \frac{f \times \Delta W(1-\cancel{x})}{2(1-\cancel{x})}$$

$$\Delta U = 3 \times 250$$

$$= 750$$

Que.) How much heat is required to supply an engine working according to $PV^2 = \text{Const.}$ with air as transmission medium If we want work out put of 500J.

$$\Delta Q = (1-x)\left(\frac{1}{1-x}+\frac{f}{2}\right)\Delta W$$

$$\Delta Q = \left(1-\frac{5}{2}\right) \times 500$$

$$\Delta Q = -\frac{3}{2} \times 500$$

$$\Delta Q = -750\,J$$

$$\Delta U = -1250\,J$$

NOTE: x is called polytropic coefficient.

Que.) A certain process is given as $Tv^2 = \text{const}$. Find the polytropic coefficient x and ΔW if ΔU is 500J. $f = 3$

$Pv^3 = \text{Const.}$

$x = 3$

$$\Delta W = \frac{2}{(1-x)f} \times \Delta U$$

$$\Delta W = \frac{500}{-3} = -\frac{500}{3}$$

GRAPHING A POLYTROPIC PROCESS ON A P-V DIAGRAM

P, v and T are never negative

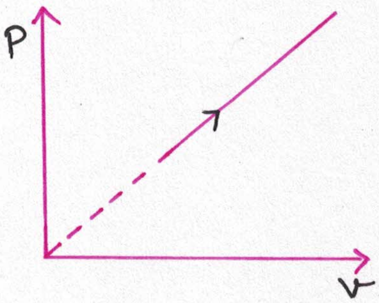
Que.) Graph the curve $Pv^{-1} = \text{Const.}$

$$P(-v)^{-2} + v^{-1}\frac{dP}{dv} = 0$$

$$\frac{dP}{dv} = \frac{P}{v}$$

$$\frac{d^2P}{dv^2} = \frac{v.\,dP/dv - P}{v^2}$$

$$= \frac{vP/v - P}{v^2} = 0$$



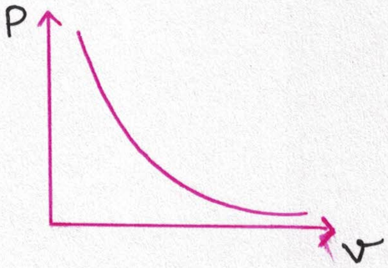
Que.) Graph the curve $Tv = \text{Const.}$

$Pv^2 = \text{const.}$

$$P \times 2v + v^2\frac{dP}{dv} = 0$$

$$\frac{dP}{dv} = -\frac{2P}{v}$$

$$\frac{d^2P}{dv^2} = -2\left(\frac{v.\,dP/dv - P}{v^2}\right)$$



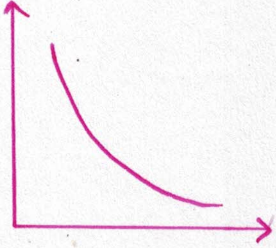
Que.) Graph the curve for $PT = \text{Const.}$

$Pv^{1/2} = \text{const.}$

$$P.\frac{1}{2\,v^{1/2}} + v^{1/2}.\frac{dP}{dv}$$

$$+\frac{dP}{dv} = -\frac{P}{2v}$$

$$\frac{d^2P}{dv^2} = -\frac{1}{2v^2}\left[v.\frac{dP}{dv} - P\right]$$



$$= -\frac{1}{2v^2}\left[v\left(-\frac{P}{2v}\right) - P\right]$$

+ve

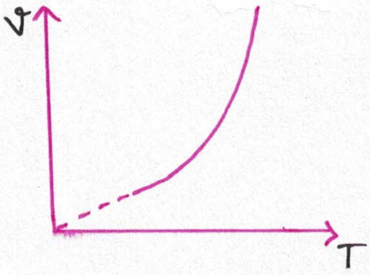
Que.) Graph v-T curve for PT = const.

$$T^2 v^{-1} = \text{const.}$$

$$T v^{-1/2} = \text{const.}$$

$$v T^{-2} = \text{const.}$$

(parabola)



$$\ln v - 2\ln T = \ln C$$

$$\frac{1}{v}\frac{dv}{dT} - 2 = 0$$

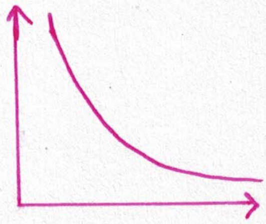
$$\frac{dv}{dT} = 2v$$

Similarly find its curvature.

Que.) Draw ρ-T curve for PT = Const.

$$\rho T^2 = \text{Const.}$$

(Same as first que)

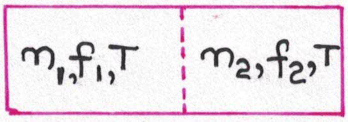


EFFECTIVE DEGREES OF FREEDOM OF A MIXTURE OF GASES

$$U = \frac{nf}{2}RT$$

$$\frac{n_1 f_1 RT}{2} + \frac{n_2 f_2 RT}{2} = (n_1 + n_2)\frac{f_{eff.}}{2}.RT$$

$$f_{eff.} = \frac{n_1 f_1 + n_2 f_1}{n_1 + n_2}$$



FIRST LAW ANALYSIS OF VARIOUS STANDARD PROCESSES

ISOBARIC PROCESS

$$C_x = \frac{R}{1-x} + \frac{f}{2}R$$

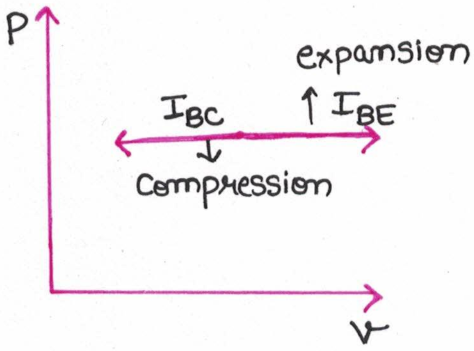
$$C_p = \frac{f}{2}R + R$$

$$C_p = \left(1+\frac{f}{2}\right)R$$

$$\Delta Q = nC_p\Delta T$$

$$\Delta U = nC_v\Delta T$$

$$\Delta W = nR\Delta T$$



ISOCHLORIC PROCESS

$$C_x = \frac{R}{1-x} + \frac{f}{2}R$$

$$C_v = \frac{f}{2}R$$

$$\Delta Q = nC_v\Delta T$$

$$\Delta U = nC_v\Delta T$$

$$\Delta W = 0$$

POSSION'S RATIO (γ)

The ratio of specific heats at constant pressure to that of constant volume is called γ.

$$\gamma = \frac{C_p}{C_v}$$

$$\gamma = \frac{(1+f/2)R}{f/2\,R}$$

$$\boxed{\gamma = 1+\frac{2}{f}}$$

$$\gamma_{mon} = 1+\frac{2}{3} = \frac{5}{3}$$

$$\gamma_{dia} = 1+\frac{2}{5} = \frac{7}{5}$$

$$\gamma_{poly} = 1+\frac{2}{6} = \frac{4}{3}$$

Cp & Cv IN TERMS OF γ

$$\frac{C_p}{C_v} = \gamma \quad \text{and} \quad C_p - C_v = \gamma$$

$$C_v = \frac{R}{\gamma - 1} \quad \text{and} \quad C_p = \frac{\gamma R}{\gamma - 1}$$

$$\therefore \boxed{C_x = \frac{R}{1-x} + \frac{R}{\gamma - 1}}$$

ISOTHERMAL

$$C_x = \frac{R}{1-x} + \frac{f}{2}R$$

$$C_T = \infty$$

$$\Delta u = \frac{nf}{2}R\Delta T = 0$$

$$dw = P.dv$$

$$P = \frac{nRT}{v}$$

$$\int dw = nRT\int_{v_i}^{v_f}\frac{dv}{v}$$

$$\boxed{\begin{aligned}\Delta w &= \Delta Q \\ &= nRT\ \ln\left(\frac{v_f}{v_i}\right)\end{aligned}}$$

$$P_f v_f = v_i P_i$$

$$\Delta w = \Delta Q = nRT \underset{\displaystyle P_i v_i \text{ or } P_f v_f}{\underset{\downarrow}{\ln}}\left(\frac{P_i}{P_f}\right)$$

ADIABATIC PROCESS (ISENTROPIC PROCESS)

$$dQ = 0 = du + dw$$

$$du = -dw$$

$$\frac{n.R}{\gamma - 1}dT = -P.dv$$

$$\frac{n.R}{\gamma - 1}dT = -nRT.\frac{dv}{v}$$

$$\frac{1}{(\gamma - 1)}\frac{dT}{T} = -\frac{dv}{v}$$

$$\frac{1}{(\gamma-1)} \ln T = -\ln v + C$$

$$T^{1/\gamma-1} . v = \text{Const.}$$

$$T . v^{\gamma-1} = \text{Const.}$$

$$\boxed{Pv^{\gamma} = \text{Const}}$$

Or

$$C = \frac{R}{1-x} + \frac{R}{\gamma-1} = 0$$

$$(dQ = 0) \quad \Rightarrow x = \gamma$$

$$\Delta U = \frac{nf}{2} R \Delta T$$

$$\Delta W = -\frac{nf}{2} R \Delta T \qquad \Delta Q = 0$$

$$Pv^{\gamma} = C$$

$$\ln P + \gamma \ln v = \ln C$$

$$\frac{1}{P} \cdot \frac{dP}{dv} = -\frac{\gamma}{v}$$

$$\frac{dP}{dv} = -\frac{\gamma P}{v}$$

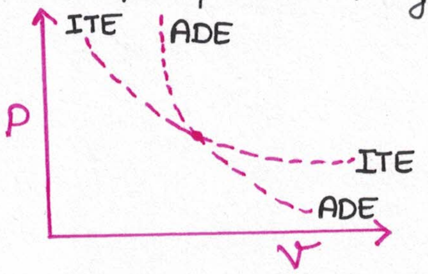
(as $\gamma > 1$, it will have more Steeper slope than isothermal process)

$$\frac{d^2P}{dv^2} = -\frac{\gamma}{v^2}\left[v\frac{dP}{dv} - P\right]$$

$$= -\frac{\gamma}{v^2}\left[-\gamma P - P\right]$$

$$\frac{d^2P}{dv^2} = \gamma(\gamma+1)\frac{P}{v^2}$$

for isothermal, replace γ by 1



area under P-v graph is work done by gas

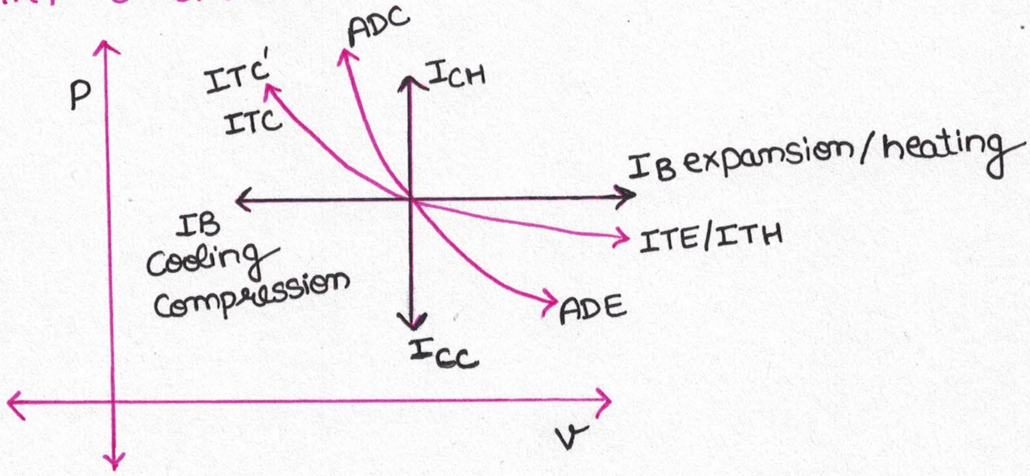
Gas expands then work done '+ve'

gas compress then work done '-ve'

For expansion : $W_{isothermal}$ is more

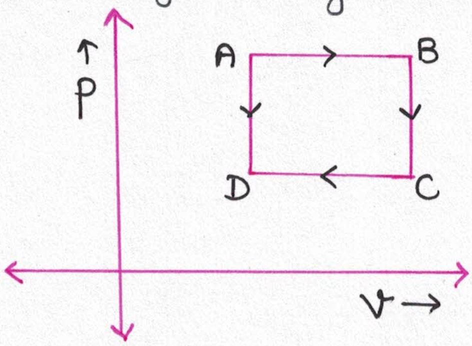
For compression : $W_{adiabatic}$ is more

SUMMARY OF GRAPH -



CYCLIC PROCESS

If a gas returns to its original state after passing through various states, we say that gas has undergone a cyclic process.



NOTE: (1) The net work done in the cyclic process is equal to the area enclosed by the cycle on a PV graph.

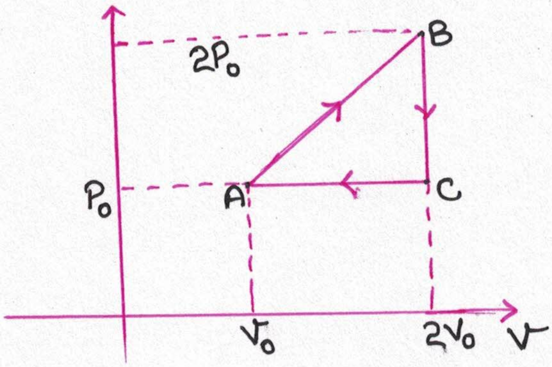
(2) The net work is positive if cycle is clockwise (engine cycle).

(3) The net work is negative if cycle is anti-clockwise (refrigerate cycle)

$\Delta U = 0$ in a cyclic process.

EFFECIENCY OF A CYCLIC PROCESS

$$\eta = \frac{\sum \Delta w}{\sum \Delta Q_+}$$ → positive heat given



An engine based on a diatomic gas undergoes a cycle as shown in the figure.

(a) Specify pressure, volume & temperature at A, B & C

(b) Calculate ΔQ, ΔU & Δw for each of processes.

(c) Calculate efficiency of engine.

Process	P	V	T
A	P_0	V_0	$\frac{P_0V_0}{nR}$
B	$2P_0$	$2V_0$	$\frac{4P_0V_0}{nR}$
C	P_0	$2V_0$	$\frac{2P_0V_0}{nR}$

A → B → $PV^{-1} = C$

$$\Delta U = \frac{nfR\Delta T}{2} = \frac{5}{2}\frac{\cancel{nR}\,3P_0V_0}{\cancel{nR}} = \frac{15P_0V_0}{2}$$

$$\Delta w = \frac{nR\Delta T}{1-n} = \frac{3P_0V_0}{2}$$

B → C

$$\Delta U = \frac{nfR\Delta T}{2}$$

$$= \frac{5}{2}nR\left(-\frac{2P_0V_0}{nR}\right)$$

$$= -5P_0V_0$$

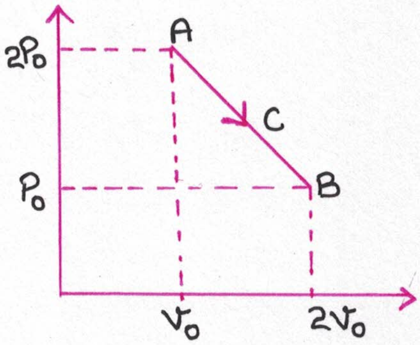
	ΔU	Δw	ΔQ
A → B	$\frac{15}{2}P_0V_0$	$\frac{3}{2}P_0V_0$	$9P_0V_0$
B → C	$-5P_0V_0$	0	$-5P_0V_0$
C → A	$-\frac{5}{2}P_0V_0$	$-P_0V_0$	$-\frac{7}{2}P_0V_0$

$\sum \Delta w = \frac{1}{2} P_0 v_0$ $\sum \Delta Q^+ = 9 P_0 v_0$

$\sum \Delta w = \frac{1}{2} P_0 v_0$

$\eta = \frac{\sum \Delta w}{\sum \Delta Q^+} = \frac{1}{18}$

Que.) A gas expands from A to B as shown on the P-v graph. Find (i) equation of the line
(ii) The volume at which temp. is max.
(iii) maximum temp.



$y - P_0 = \frac{P_0}{v_0}(x - 2v_0)$

$\frac{P}{P_0} + \frac{v}{v_0} = 3$

$\frac{nRT}{vP_0} + \frac{v}{v_0} = 3$

$\frac{nR}{P_0}\left[\frac{v\,dT/dv - T}{v^2}\right] + \frac{1}{v_0} = 0$ $\left(\frac{dT}{dv} = 0\right)$

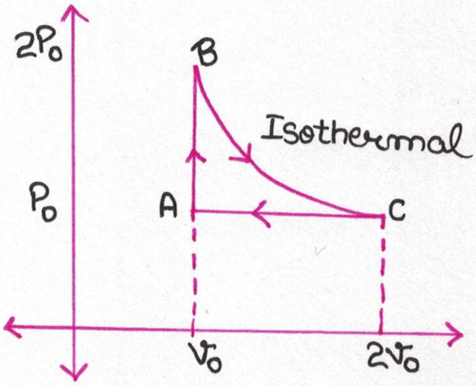
$\frac{nRT}{P_0 v^2} = \frac{1}{v_0}$

$\frac{P}{P_0 v} = \frac{1}{v_0}$

$v = \frac{3}{2} v_0$ and $P = \frac{3P_0}{2}$

$T_{max} = \frac{9}{4}\frac{P_0 v_0}{nR}$

Que.) $f = 3$. Find efficiency of a gas.



$\sum w = w_{AB} + w_{BC} + w_{CA}$
$= 0 + 2P_0 v_0 \ln(2) - P_0 v_0$
$= 0.386\, P_0 v_0$

$\sum Q^+ = Q_{AB} + Q_{BC} + Q_{CA}$ (if positive)

$= \frac{3}{2} nRT_0 + 2nRT \ln 2 \underbrace{- \frac{3}{2} nRT_0 - P_0 v_0}_{\text{this is to be ignored}}$

$$\eta = \frac{2\ln 2 - 1}{\frac{3}{2} + 2\ln 2}$$

$$= \frac{2\times 0.7 - 1}{3/2 + 2\times 0.7} = \frac{4}{29}$$

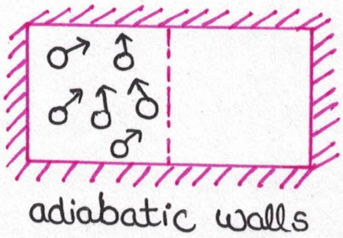
FREE EXPANSION

When a confined gas is allowed to expand in an empty space, we call the process free expansion.

$\Delta Q = 0$

$\Delta U = 0$

$\Delta W = 0$



NOTE: 1.) In free expansion the rms velocity of the molecules does not change. Further, most of the times free expansion is brought about suddenly and thus immediately after expansion ΔV is assumed to be zero.

2.) Almost all sudden process of expansion or compression mean $\Delta Q = 0$ immediately afterwards.

3.) Fast Process :- Adiabatic

Slow Process :- Isothermal

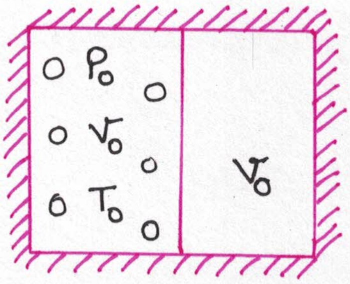
Que.) A monoatomic gas is confined to the volume V_0 while the temp. & pressure and T_0 & P_0. Its volume is doubled by free expansion.

(i) What is its new pressure.

(ii) What is its new temperature.

(iii) How much work is to be required to compress it adiabatically back to same volume.

(iv) What will be its temp. after adiabatic compression.



(i) $P' = \frac{P_0}{2}$

(ii) $T' = T_0$

(iii) $\Delta w = \frac{P_f V_f - P_i V_i}{(1-x)}$

$$P_f V_0^{\gamma} = \left(\frac{P_0}{2}\right)(2V_0)^{\gamma}$$

$$P_f = \frac{P_0}{2} \times 2^{\gamma}$$

$$P_f = \frac{P_0}{2} \times 2^{5/3}$$

$$\Delta w = \frac{P_0/2 \times 2^{5/3} V_0 - P_0/2 \times 2V_0}{-2/3}$$

$$\Delta w = \frac{3}{4} P_0 V_0 \left[2 - 2^{5/3}\right]$$

(iv) $T_0 (2V_0)^{\gamma-1} = T V_0^{\gamma-1}$

$$T = T_0 2^{\gamma-1}$$

$$T = T_0 2^{2/5}$$

$$T = T_0 2^{2/3}$$

According to 1st law :

$$\underset{\substack{\text{K.E \& P.E. of} \\ \text{the system}}}{\Delta q = \Delta U} + \Delta w \rightarrow$$ work done by the system on surroundings

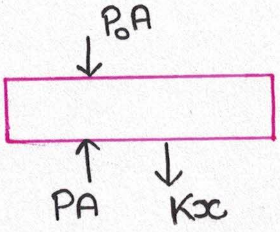
Here, we have not included work done by surroundings because it is equal to change in Kinetic energy of the system which is already included in ΔU.

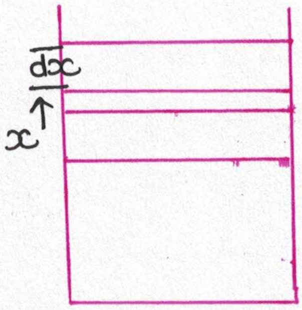
Que.) A gas confined under a massless system at atmospheric pressure, is expanded to double its volume by heating. Considering only the gas as a system, find the ΔU, ΔQ and Δw in the process.

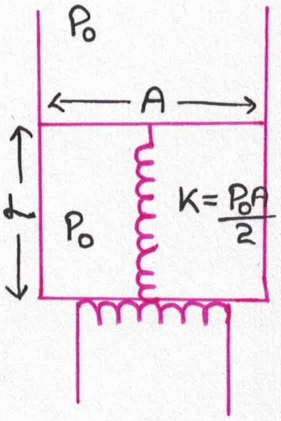
$$PA = P_0 A + Kx$$

$$P = \frac{P_0 A + Kx}{A}$$

$$dw = P.dV$$







$$dw = \frac{P_0A + Kx}{A} \times A.dx$$

$$\int_0^{\Delta w} dw = \int_0^{l} P_0A + Kx \, .dx$$

$$= P.Al + \frac{Kl^2}{2}$$

$$\Delta w = \frac{3}{2} P_0 Al$$

$$P_f = P_0 + \frac{Kl}{A}$$

$$= 2P_0$$

$$V_f = 2V_0$$

$$U = \frac{nf}{2} R\Delta T$$

$$\Delta U = \frac{3}{2}\left[4P_0V_0 - P_0V_0\right]$$

$$\Delta U = \frac{9}{2} P_0V_0 = \frac{9}{2} P.Al$$

$$U_{spring} = \frac{1}{2}Kl^2$$

$$= \frac{P_0Al}{2}$$

Que.) Repeat the previous problem if your system is spring + gas + piston.

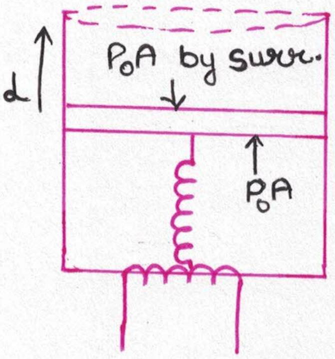
$$\Delta w = P_0Al$$

$$\Delta U = \frac{9}{2}P_0Al + \frac{1}{2}Kl^2$$

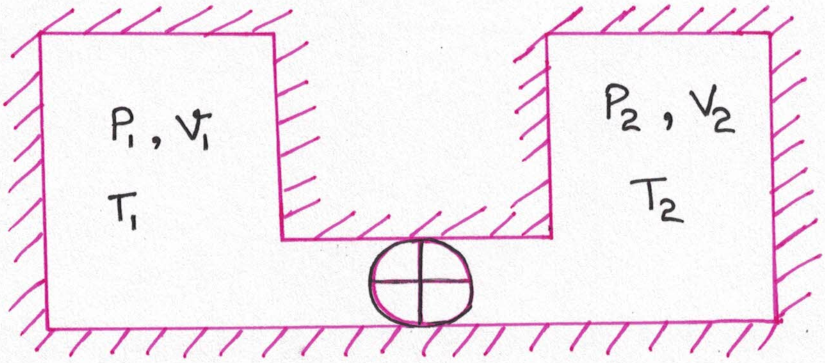
$$= \frac{9}{2}P_0Al + \frac{P_0}{2}Al$$

$$= 5P_0Al$$

$$\Delta Q = 6P_0Al$$



Que.) Walls are adiabatic and the gas is monoatomic. Find final pressure and temp. when the valve is open.



$$U_i \text{ (initially)} = \frac{n_1 f R T_1}{2} + \frac{n_2 f R T_2}{2}$$

$$U_i = \frac{P_1 V_1 f}{2} + \frac{P_2 V_2 f}{2}$$

$$U_f = \frac{P(V_1+V_2) f}{2}$$

as no heat is supplied & no work is done $\Delta U = 0$

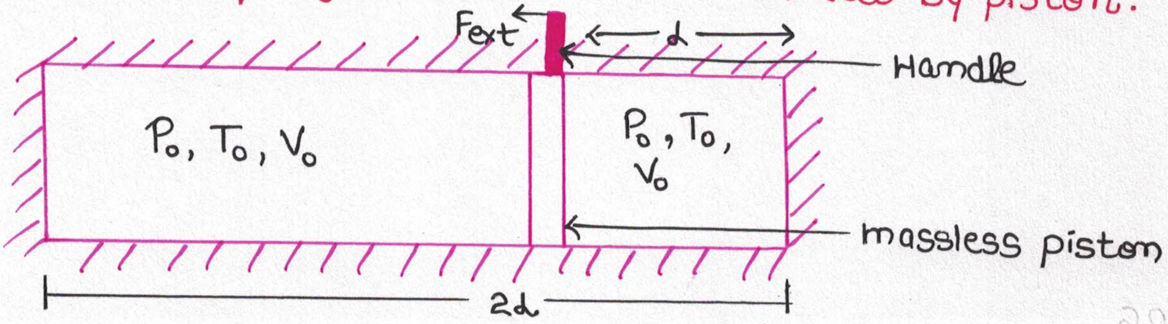
$$P = \frac{P_1 V_1 + P_2 V_2}{V_1 + V_2}$$

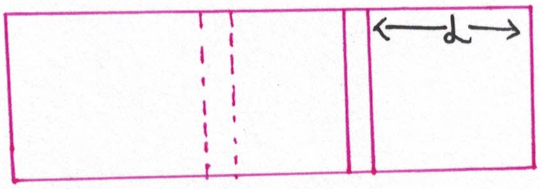
$$T = \frac{P(V_1+V_2)}{R \times n}$$

$$T = \frac{P_1 V_1 + P_2 V_2}{\cancel{R} \times \left(\frac{P_1 V_1}{T_1 \cancel{R}} + \frac{P_2 V_2}{\cancel{R} T_2}\right)}$$

$$T = \frac{(P_1 V_1 + P_2 V_2) T_1 T_2}{P_1 V_1 T_2 + P_2 V_2 T_1}$$

Que.) Piston is perfectly conducting. A chamber has a partioning massless system as shown in the fig. Now by using an Fext, the piston is moved so as to equalise the volumes on both sides. Considering the gas only as the system Form differential eqn. of T and distance covered by piston.





$\Delta Q = 0$ (System is both the gases)

$dw + dU = 0$

$dw = -dU$

$dw = -\frac{nf}{2} RT$ —①

$$\frac{P_0 V_0}{T_0} = \frac{P_R (V_0 + Ax)}{T}$$

$$P_R = \frac{P_0 V_0 T}{T_0 (V_0 + Ax)}$$

$$P_L = \frac{P_0 V_0 T}{T_0 (V_0 - Ax)}$$

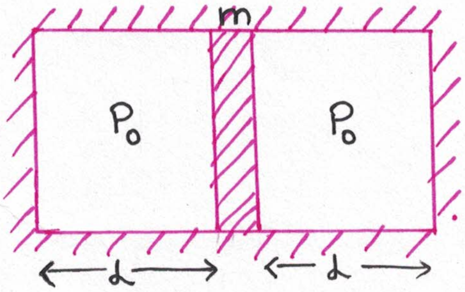
$$(P_R - P_L)\, A.dx = \frac{nf}{2} R\, dT$$

@ $x = 0$, $T = T_0$

and @ $x = x$, $T = T$

$$n = \frac{P_0 V_0}{T_0 R}$$

Que.) Find the time period of oscillation of piston (adiabatic & massive). Make suitable approximation.



$f = 3$

$$(P_L)(\ell A - Ax)^{\gamma} = P_0 (A\ell)^{\gamma}$$

$$P_L = \frac{P_0 (A\ell)^{\gamma}}{(\ell A - Ax)^{\gamma}}$$

$$P_L = \frac{P_0}{\left(1 - \frac{x}{\ell}\right)^{\gamma}}$$

$$P_R = \frac{P_0}{\left(1 + \frac{x}{\ell}\right)^{\gamma}}$$

As x is very, very small

$$P_L = P_0\left(1+\frac{\gamma x}{L}\right) \quad \text{and} \quad P_R = P_0\left(1-\frac{\gamma x}{L}\right)$$

$$(P_R - P_L)A = m\frac{d^2x}{dt^2}$$

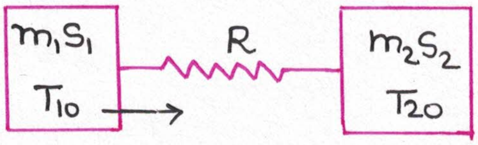
$$P_0\left(\cancel{1}-\frac{\gamma x}{L} - \cancel{1} - \frac{\gamma x}{L}\right)A = m\frac{d^2x}{dt^2}$$

$$-\frac{2P_0A\gamma x}{L} = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\frac{2P_0A\gamma}{Lm}x$$

$$T = 2\pi\sqrt{\frac{2m}{2P_0A\gamma}}$$

Que.) Find the temp. difference b/w the reservoirs as a function of time.



$$\frac{dQ}{dt} = \frac{T_1-T_2}{R} = \frac{m_2S_2dT_2}{dt} \quad \text{—①}$$

$$\frac{dQ}{dt} = \frac{T_1-T_2}{R} = -\frac{m_1S_1dT_1}{dt} \quad \text{—②}$$

$$\frac{①}{m_2S_2} + \frac{②}{m_1S_1}$$

$$\left(\frac{T_1-T_2}{R}\right)\left(\frac{1}{m_2S_2}+\frac{1}{m_1S_1}\right) = \frac{d}{dt}(T_2-T_1)$$

Put $(T_1-T_2) = x$

$$\frac{x}{R}\left(\frac{m_1S_1+m_2S_2}{m_1S_1\cdot m_2S_2}\right) = -\frac{dx}{dt}$$

$$\int_{T_{10}-T_{20}}^{x}\frac{dx}{x} = -\int_0^t\left(\frac{m_1S_1+m_2S_2}{m_1S_1\cdot m_2S_2}\right)dt$$